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LETTER TO THE EDITORS

PREDICTION OF WALL TEMPERATURES FOR INTERNAL LAMINAR HEAT TRANSFER

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IN A RECENT paper, Worsøe-Schmidt and Leppert have presented the first complete solution to the modified Graetz problem including strong variation of gas properties [1]. Within a practical range of heating rates, the predicted Nusselt numbers differ from the constant properties solution by less than 20 per cent; this result is shown to be consistent with the available experimental data which was obtained with smaller, but significant, heating rates. The authors are too modest. If another definition of the local Graetz number is applied, the agreement is even more striking.

Figure 1 shows the numerical results of Worsøe-Schmidt and Leppert plotted in terms of local bulk parameters. In particular, the Graetz number is based on the local Reynolds number, defined as $4m/(\pi D\mu_{x,m})$, rather than the inlet Reynolds number as in their paper*. Included for comparison is the constant properties prediction of Siegel, Sparrow and Hallman [2]. It is seen that the difference between the most extreme heating rate and the isothermal prediction falls to within 10 per cent at $Gz_{x,m} = 85$, which is still well within the thermal entrance region. Thus, one may conclude there is no significant difference for $Gz_{x,m} \gtrsim 80$ when calculated in this manner. This is almost exactly the conclusion drawn by Kays and Nicoll from their data with a slightly decreasing wall heat-flux boundary condition [3].

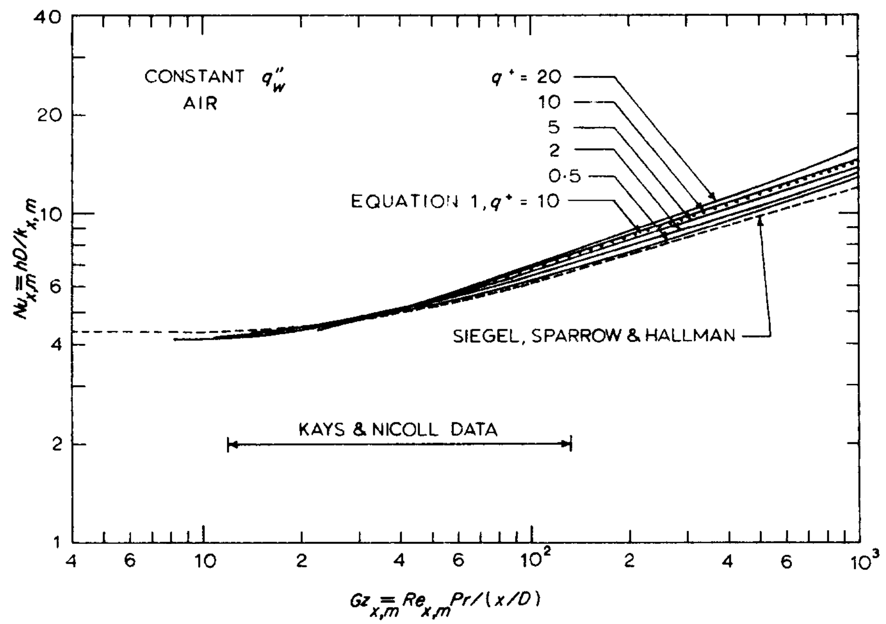

FIG. 1. Local heat-transfer predictions based on Graetz number evaluated at local Reynolds number. Extent of data for highest heating rate, $q^+ \approx 6{\cdot}6$, of Kays and Nicoll shown for comparison [2].

* The subscript m refers to evaluation of fluid properties at the bulk temperature.

Data obtained to date have been inconclusive in the range of the thermal entry where the difference from constant properties predictions becomes significant; axial conduction, axial radiation and other experimental problems hinder an accurate determination of the heat-flux distribution. However, by using the local Graetz number it is possible to obtain a moderately simple analytical correlation for the local Nusselt number in this region:

$$\frac{Nu_{x,m}}{Nu_{\text{iso}}} = \{1 - 0{\cdot}00125(q^+)^{3/2}\}\, Gz_{x,m}^{0{\cdot}0025q^+} \qquad (1)$$

The parameter q^+ is defined in the manner of Worsøe-Schmidt and Leppert as $r_0 q''_w/(k_0 T_0)$. This fit to their prediction was determined for the range

$$100 < Gz_{x,m} < 1000 \qquad \text{and} \qquad q^+ \leqslant 20.$$

Axial variation of the Prandtl number was neglected.

If the proposed correlation is used to a Graetz number of 60 and the isothermal result is used further downstream, the Nusselt number prediction should agree with the tabulated values of the complete analysis to within about 7 per cent. For comparison purposes, equation (1) is plotted as a dotted line on the figure for $q^+ = 10$ and

$$Nu_{\text{iso}} = 15{\cdot}8\, Gz^{0{\cdot}3} \qquad (2)$$

Equation (2) is an extrapolation of the results of Worsøe-Schmidt and Leppert to $q^+ = 0$ for the Graetz number range noted above. If reference [2] is used to predict Nu_{iso}, the dotted line is about 4 per cent lower for $Gz_{x,m} \gtrsim 100$.

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